

Experiment 2: Periodic Signals and the Fourier Series

1 Introduction

In Experiment 1 we found that a sinusoid of frequency f_0 transforms to impulses at $+f_0$ and $-f_0$. In this, the second experiment, we will investigate the Fourier series for periodic signals. The mathematical aspect of the signals will be explained followed by an experiment with the spectrum analyzer allowing you reinforce these important concepts.

2 The Fourier Series

Fourier discovered that any periodic signal could be represented as a sum of sinusoids. A waveform with periodicity T seconds and frequency $f_0 = \frac{1}{T}$ can be represented as the sum of sinusoids with frequencies that are integer multiples of f_0 . ($f_0, 2f_0, 3f_0$, etc.) Mathematically we would write:

$$x(t) = \sum_{n=0}^{\infty} [a_n \cos(n2\pi f_0 t) + b_n \sin(n2\pi f_0 t)] \quad (1)$$

The Fourier series can also be written in its more convenient but somewhat less intuitive form:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn2\pi f_0 t} \quad (2)$$

The transform of the Fourier series can be found to be:

$$X(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_0) \quad (3)$$

$X(f)$ is a summation of impulses. These impulses may only occur at integer multiples (harmonics) of the fundamental frequency f_0 .

We will now observe these impulses by inputting several different periodic signals from the function generator to the spectrum analyzer. Remember, any real signal, $x(t)$, has a transform magnitude, $|X(f)|$, with even symmetry. Because spectrum analyzers are designed to show the transform magnitude, if we were to look at negative frequencies, we would observe a mirror image of the positive impulses on the negative frequency axis. The following steps will guide you through the experiment.

3 Fourier Series of a Square Wave

This section will lead you step by step through the process of setting up the Stanford Research SR770 FFT Network/Spectrum Analyzer to view $|X(f)|$ in real-time.

1. **Generate a square wave:** Use a function generator to create a square wave of **500 Hz**.
2. **Connect** the function generator to the spectrum analyzer's **A** Input using a BNC to BNC cable.
3. Set up the spectrum analyzer:
 - Turn the analyzer on while holding down the [←] (backspace) key. Continue holding down backspace until the Calibrating Offset message appears on the display.
 - Once the Calibrating Offset message has cleared, press the **AUTO RANGE** button from the **ENTRY** group.
 - Hit the **MEAS** (measure) button from the **MENU** group. Hit the **Display Menu:** softkey. Next, to change the display from log to linear scale, press the **Linear Mag** softkey.
 - Press **AUTO SCALE** from the **ENTRY** group.
 - Now we'll adjust the frequency range in order to clearly display our signal's spectrum. Press **FREQ** from the **MENU** group. Press the **Span** softkey, and then use the knob or the **SPAN** ↓ button to adjust the span to **6.25 kHz**.
 - From the **MARKER** group (below the center knob), press **MAX/MIN**. This centers the marker region around the largest data point on the graph. The marker should now be on the 500 Hz impulse. The marker readout above the graph displays frequency and amplitude of the signal.
4. At this point you should see a large impulse at 500 Hz followed by impulses of decreasing size at 1500 Hz, 2500 Hz, 3500 Hz, etc. The height of each of these impulses corresponds to the magnitude of each coefficient, c_n , in the Fourier series for our square wave. The first impulse at 500 Hz corresponds to $n = 1$ and is has a height proportional to the magnitude of c_1 .

Record the magnitude, in mV, of the first five impulses for $n = 1, 3, 5, 7$, and 9 . To accomplish the measurements use the on-screen marker. Hit **MAX/MIN** again to put the marker at the peak of the impulse at 500 Hz ($n = 1$). Use the knob to move the marker and measure the magnitude at each peak.

5. An explanation for the particular pattern of coefficients/impulses that you are witnessing can be found by solving the following Fourier series analysis equation where T is the time duration

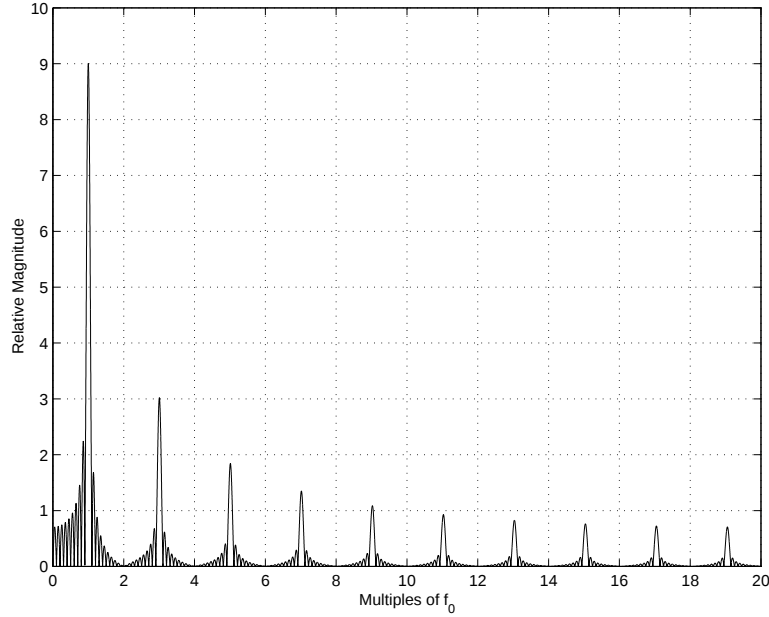


Figure 1: Transform of square wave, impulses from $n = 1$ to $n = 19$

of one period of our signal and f_0 is the fundamental frequency, $\frac{1}{T}$.

$$c_n = \frac{1}{T} \int_T x(t) e^{-jn2\pi f_0 t} dt \quad (4)$$

Use equation (4) to derive a general solution for the coefficients, c_n , of a square wave with amplitude A. (Example 4.5 in Dr. Jackson's book may be helpful.)

6. Use the Matlab FFT (Fast Fourier Transform) function to find the transform of a square wave. Plot your results. The graph in Figure 1 was generated by taking the FFT a square wave 10 periods in length. Each period consisted of 40 samples, 20 positive, and 20 negative.

The results obtained with the spectrum analyzer should be supported by both Matlab and the derived equations. With the impulse at $n = 1$ as a reference, you should find that the magnitude of each successive impulse decreases as $\frac{1}{n}$. Verify that, for the measurements taken from the spectrum analyzer, the impulse at $n = 3$ is $\frac{1}{3}$ the size of the first impulse, the impulse at $n = 5$ is $\frac{1}{5}$ the size, etc.

4 Fourier Series of a Triangular Wave

1. Configure the function generator to produce a **triangular wave** of **500 Hz**.
2. Set up the spectrum analyzer in the same configuration that was used for analyzing the square wave.

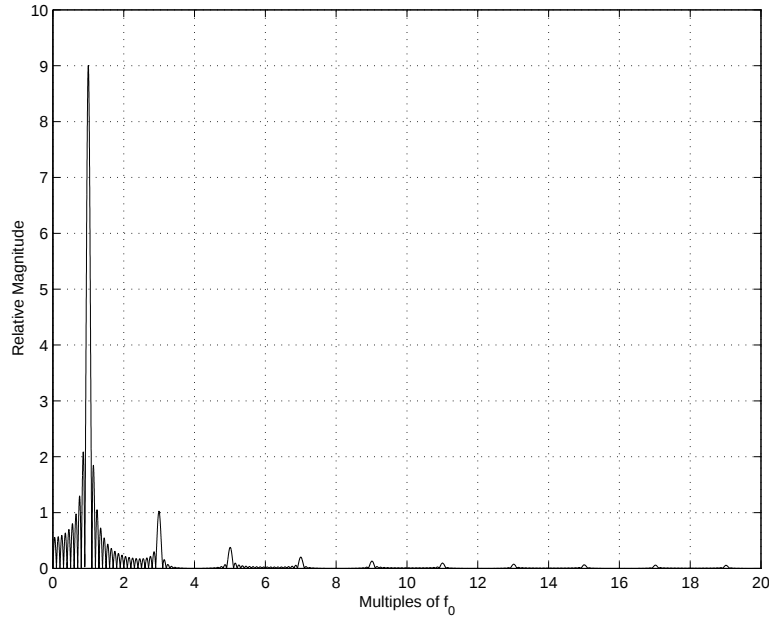


Figure 2: Transform of triangular wave, impulses from $n = 1$ to $n = 19$

3. Once again, you should see a large impulse at 500 Hz followed by impulses of decreasing size at 1500 Hz, 2500 Hz, 3500 Hz, etc. Notice that the impulses decrease in size much more rapidly than the impulses associated with the square wave. Also notice that the impulses are only at odd values of n . Why might this be? Record the magnitude, in mV, of the first four impulses for $n = 1, 3, 5$, and 7 .
4. Use the Matlab FFT function to find the transform of a triangular wave. Plot your results. The graph in Figure 2 was generated by taking the FFT a triangular wave 10 periods in length. Each period consisted of 40 samples.
5. Based on your measurements and Matlab experiments, determine the rate at which the magnitude of the coefficients, c_n , decrease for a triangular wave. Remember that the coefficients of a square wave decrease as $\frac{1}{n}$.

5 Fourier Series of a Sawtooth Wave (repeating ramp)

1. Configure the function generator to produce a **sawtooth wave** of **500 Hz**.
2. Set up the spectrum analyzer in the same configuration that was used for analyzing the square and triangular waves.
3. This time, you should see a large impulse at 500 Hz followed by impulses of decreasing size at 1000 Hz, 1500 Hz, 2000 Hz, etc. Why are there both odd and even harmonics for this signal,

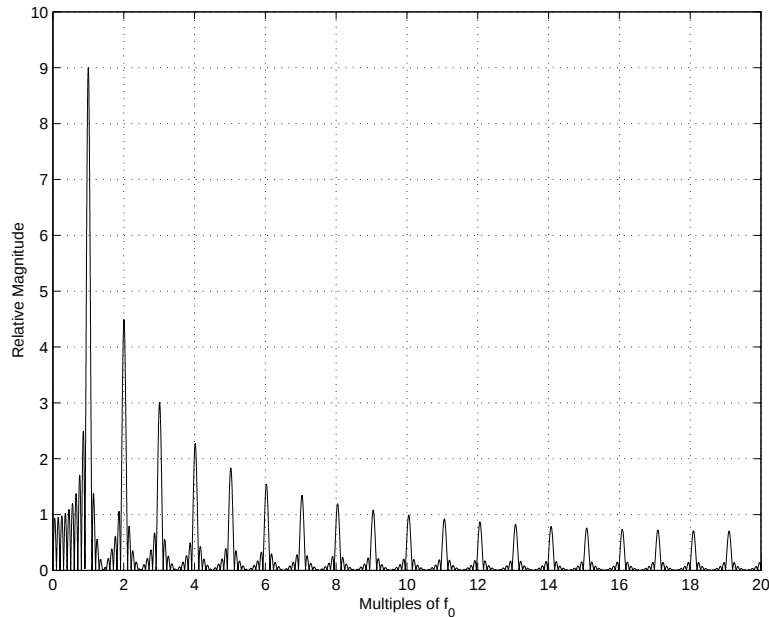


Figure 3: Transform of sawtooth wave, impulses from $n = 1$ to $n = 19$

but not for the previous two signals? (See Dr. Jackson's problem 4.6) Record the magnitude, in mV, of the first nine impulses for $n = 1$ to 9.

4. Use the Matlab FFT function to find the transform of a sawtooth wave. Plot your results. The graph in Figure 3 was generated by taking the FFT a sawtooth wave 10 periods in length. Each period consisted of 40 samples.
5. Based on your measurements and Matlab experiments, determine the rate at which the magnitude of the coefficients, c_n , decrease for a sawtooth wave.

6 Fourier Series Approximation of a Signal

The Fourier Series is an infinite series, but for most periodic signals we may only need a few harmonics to get a good approximation. There is a great java applet on the Johns Hopkins University website that allows the user to do exactly this. Follow the **Fourier Series Approximation** link from the ELE 436 webpage. Once the applet has loaded, sketch a square wave and print out a plot of the 20 coefficient approximation. **NOTE:** The square wave is not one of the standard waveforms with a corresponding button. You must actually click the mouse in the waveform area and drag to "draw" the square wave. Click the table button and then copy the list of coefficients to your favorite word processing program. Include a printout of your waveform and the list of coefficients in your report. Check to make sure that your results are reasonable.